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The extended McEvily model for fatigue crack growth analysis of metal structures

Yifei Wang [a,*], Weicheng Cui [a,b], Xiaoyuan Wu [c], Fang Wang [a], Xiaoping Huang [a,d]

[a] State Key Laboratory of Ocean Engineering, Shanghai Jiao Tong University, Shanghai 200030, China
[b] China Ship Scientific Research Center, Wuxi 214082, China
[c] Shanghai Waigaoqiao Shipbuilding Company Limited, Shanghai 200137, China
[d] Centre for Ships and Ocean Structures, Norwegian University of Science and Technology, Otto Nielsens v 10, N-7491 Trondheim, Norway

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Abstract

An extended McEvily model for fatigue crack growth analysis of metal structures is proposed in this paper. In comparing with our previous work, the extension is mainly concerned with the following two aspects: (1) the slope of the fatigue crack growth rate curve is regarded as a variable rather than a fixed value for different materials; (2) both the maximum stress intensity factor at the crack opening level, $K_{op,max}$ and the effective stress intensity factor range at the threshold level, $\Delta K_{eff,th}$ are functions of load ratio, R and they are determined by the curve fitting method. Results indicate that the value of $K_{op,max}$ tends to decrease slightly as load ratio increases where crack closure is experimentally detected. According to the present data obtained through the nonlinear least squares fitting method and discussions on the experimental results in the published literature, the parameter $\Delta K_{eff,th}$ increases with increasing load ratios where crack closure exists and decreases at high load ratios where the experimental data are closure free. In this paper, all the parameters in the extended McEvily model are assumed to be unknown in advance and they are estimated through the curve fitting method based on the experimental data. The method is also put forward to determine material constants in the crack growth rate law based on the fitting parameters under different load ratios. Comparison between the predicted results and the corresponding experimental data with different load ratios reveals that the extended McEvily model can account for the load ratio effect reasonably well. Based on our previous work this model can also be expected to explain other fatigue phenomena.

Keywords: Fatigue crack growth; Threshold; Effective stress intensity factor range; Load ratio; Crack closure

1. Introduction

Marine structures such as ships and offshore platforms are frequently subjected to complex loading histories and one of the most significant failure modes is fatigue. Marine structures are mostly made of metals. Though fatigue of metals and metal structures has been studied for more than 160 years [1], mechanisms of metal fatigue have not been fully understood [2]. Generally speaking, two different theories for predicting the fatigue life of metal structures have been developed [3,4]. One is the cumulative fatigue damage theory based on S–N (or ε–N) curves and the other is the fatigue crack propagation theory based on the crack growth rate curve. Large scatter always appears in the predicted fatigue lives, because the cumulative fatigue damage theory cannot account for the effects of initial crack size and load sequence [5]. The fatigue crack propagation theory can overcome these difficulties. Hence, much progress has been made for the fatigue crack propagation theory since the famous Paris equation [6] was proposed.

As one of the fatigue crack growth models, McEvily model [7–12] cannot only account for the effects of initial crack size and load sequence, but also explain various other phenomena of metal fatigue observed in tests. McEvily model is valid for both physically short crack and

* Corresponding author. Tel.: +86 021 62932056.
E-mail address: xh-wyf@sjtu.edu.cn (Y. Wang).

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macroscopically long crack [11,13]. The model has been successfully applied to many fatigue problems such as fatigue problem under classical two-step fatigue loading [14], fatigue problem under multiple two-step fatigue loading [15], fatigue problem with overload [16] and fatigue problem under biaxial loading [17]. The model shows promising capability and is worthy of being further studied. Cui and Huang [18] suggested a general fatigue crack growth model with nine parameters by extending McEvily model. The general fatigue crack growth model is valid from near threshold region to unstable fracture region. Zhou and Cui [19] proposed a method of estimating the nine parameters in the general model according to the existing experimental data and carried out sensitivity analyses for the nine model parameters. Wang et al. [20] studied the effect of parameter reflecting crack closure development on the fatigue crack growth rate through the nonlinear least squares fitting method. The results were not entirely the same as that of Zhou and Cui [19]. Results showed that for macroscopic cracks the parameter reflecting crack closure development has little effect on the fatigue crack growth rate when it exceeds a certain value. However, when the fatigue crack propagation is in the short crack region, the parameter has significant effect on the fatigue crack growth rate. Li et al. [21] pointed out that McEvily model with the fixed slope of two is not in agreement with many experimental results. It is strongly recommended that the slope should be a variable.

According to these previous studies, further extension to McEvily model is proposed in this paper. The extended McEvily model is valid from near threshold region to unstable fracture region and the slope of the fatigue crack growth rate is viewed as a variable. As mentioned by McEvily et al. [10], both the maximum stress intensity factor at the crack opening level for a macroscopic crack, $K_{\mathrm{op,max}}$ and the effective stress intensity factor range at the threshold level, $\Delta K_{\mathrm{eff,th}}$ are functions of load ratio, R. Two equations representing $K_{\mathrm{op,max}}$ and $\Delta K_{\mathrm{eff,th}}$ using load ratio R, respectively will be presented based on the experimental data. The process is accordingly shown to determine the parameters in the extended McEvily model. Finally, the predicted results based on the obtained parameters are compared with the corresponding experimental data under different load ratios.

2. The extended McEvily model for fatigue crack growth analysis

2.1. Modified constitutive relation by McEvily and his co-workers

The modified linear elastic fracture mechanics approach is based on the following constitutive relation:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = A(\Delta K_{\mathrm{eff}} - \Delta K_{\mathrm{eff,th}})^2 \tag{1}$$

where a is the crack length; N is the number of load cycles; A is a material- and environmentally-sensitive constant of dimensions $(\mathrm{MPa})^{-2}$; ΔK_{eff} is the range of the effective stress intensity factor which is defined as

$$\Delta K_{\mathrm{eff}} = K_{\max} - K_{\mathrm{op}} \tag{2}$$

where $K_{\max}$ is the maximum stress intensity factor in a loading cycle and K_{op} is the stress intensity factor at the crack opening level. $\Delta K_{\mathrm{eff,th}}$ in Eq. (1) is the effective stress intensity factor range at the threshold level as mentioned above. Eq. (1) has been shown to be valid for a wide range of alloys [10].

In order to use Eq. (1) in the analyses of a range of topics such as (1) anomalous fatigue crack growth behavior; (2) small crack behavior; (3) mean stress effect; and (4) load sequence effect, McEvily and his co-workers [10–13] have introduced some modifications to Eq. (1) to account for (a) the elastic–plastic behavior of small cracks; (b) the variation of the crack closure level; and (c) the transition from the threshold level to the endurance limit as a controlling parameter in the small crack growth regime.

The modified constitutive relation for fatigue crack growth has been expressed as follows [9–11]

$$\frac{\mathrm{d}a}{\mathrm{d}N} = AM^2 \tag{3}$$

$$M = K_{\max}(1-R) - (1-\mathrm{e}^{-ka})(K_{\mathrm{op,max}} - RK_{\max}) - \Delta K_{\mathrm{eff,th}} \tag{4}$$

$$K_{\max} = \sqrt{\pi r_{\mathrm{e}}\left(\sec\frac{\pi}{2}\frac{\sigma_{\max}}{\sigma_{\mathrm{Y}}}+1\right)}\left(1+Y(a)\sqrt{\frac{a}{2r_{\mathrm{e}}}}\right)\sigma_{\max} \tag{5}$$

where r_{e} is the size of an inherent flaw, a parameter whose magnitude is of the order of several microns in length [10]; σ_{Y} is the yield stress of the material, MPa; $\sigma_{\max}$ is the maximum stress in a loading cycle; R is the stress ratio; $Y(a)$ is a geometrical factor; k is a material constant which reflects the rate of crack closure development with crack advance.

2.2. The general constitutive relation by Cui and Huang [18]

The modified constitutive relation for fatigue crack growth proposed by McEvily and his co-workers is further generalized in the following three aspects in Ref. [18]:

(1) To introduce an unstable fracture condition into the crack growth rate curve in order to cover the whole fatigue crack propagation regimes.
(2) To define a 'virtual strength' to replace the yield stress in order for the constitutive relation to be applicable from 'crack-free' plain specimen to cracked body and from fatigue limit to ultimate strength.
(3) To introduce an overload/underload parameter for modeling the overload retardation and underload acceleration.

Thus the general constitutive relation for fatigue crack growth can be described as the following equations

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \frac{AM^2}{1-\left(\frac{K_{\max}}{K_{\mathrm{c}}}\right)^n} \tag{6}$$

$$M = K_{\max}(1-R) - (1-e^{-ka})(K_{\mathrm{op,max}} - RK_{\max}) - \Delta K_{\mathrm{eff,th}} \tag{7}$$

$$K_{\max} = \sqrt{\pi r_e\left(\sec\frac{\pi}{2}\frac{\sigma_{\max}}{\sigma_V}+1\right)}\left(1+Y(a)\sqrt{\frac{a}{2r_e}}\right)\sigma_{\max} \tag{8}$$

where K_c is the fracture toughness of the material, (MPa)$\sqrt{\mathrm{m}}$; n is a parameter reflecting the effect of $K_{\max}/K_c$; σ_V is the virtual strength of the material derived from the following equation

$$\sqrt{\pi r_e\left(\sec\frac{\pi}{2}\frac{\sigma_u}{\sigma_V}+1\right)}\left(1+Y(r_e)\sqrt{\frac{r_e}{2r_e}}\right)\sigma_u = K_c \tag{9}$$

where σ_u is the ultimate strength of the material, MPa.

The parameter reflecting the overload retardation and underload acceleration will not be addressed in this paper. The definition of this parameter can be found in Ref. [18].

2.3. Further extension to McEvily model

It is shown in Ref. [21] that McEvily model with the fixed slope of 2 in Eq. (3) is not in agreement with many experimental results. Thus the parameter m is adopted to represent the slope of the fatigue crack growth rate curve for different materials in the extended McEvily model. It is also pointed out that both $\Delta K_{\mathrm{eff,th}}$ and $K_{\mathrm{op,max}}$ are functions of load ratio, R [10]. Then the two parameters are regarded as variables for different load ratios. However, the expressions recommended in Ref. [18] for $\Delta K_{\mathrm{eff,th}}$ (R) and $K_{\mathrm{op,max}}$ (R) were found to be restrictive and in this paper, it is suggested to determine these two relations based on the experimental data. Therefore, the present extended McEvily model for fatigue crack growth analysis can be described as follows:

$$\frac{da}{dN} = \frac{AM^m}{1-\left(\frac{K_{\max}}{K_c}\right)^n} \tag{10}$$

$$M = K_{\max}(1-R) - (1-e^{-ka})(K_{\mathrm{op,max}} - RK_{\max}) - \Delta K_{\mathrm{eff,th}} \tag{11}$$

$$K_{\max} = \sqrt{\pi r_e\left(\sec\frac{\pi}{2}\frac{\sigma_{\max}}{\sigma_V}+1\right)}\left(1+Y(a)\sqrt{\frac{a}{2r_e}}\right)\sigma_{\max} \tag{12}$$

$$K_{\mathrm{op,max}} = f_1(R) \tag{13}$$

$$\Delta K_{\mathrm{eff,th}} = f_2(R) \tag{14}$$

where m is a constant representing the slope of the corresponding fatigue crack growth rate curve; f_1 (R) and f_2 (R) are functions of $K_{\mathrm{op,max}}$ and $\Delta K_{\mathrm{eff,th}}$ against load ratio R, respectively. In this extended McEvily model, parameters A, m, n, k, K_c, r_e and σ_V are regarded as material constants. Since there are so many parameters to be determined, in order to facilitate the determination, a three-step method is used. In the first step, for each R value, a set of (A, m, n, k, K_c, r_e, σ_V, $K_{\mathrm{op,max}}$ and $\Delta K_{\mathrm{eff,th}}$) are determined based on the nonlinear least squares curve fitting method. Then, for the values of $K_{\mathrm{op,max}}$ and $\Delta K_{\mathrm{eff,th}}$ under different R, two relations of $K_{\mathrm{op,max}}$ (R) and $\Delta K_{\mathrm{eff,th}}$ (R) are derived. The average values for (n, k, K_c, r_e, σ_V) under different R are taken as the final material property values. In the third step, by substituting $K_{\mathrm{op,max}}$ (R) and $\Delta K_{\mathrm{eff,th}}$ (R) relations and the final values of (n, k, K_c, r_e, σ_V) into the extended McEvily model, the best values of A and m are redetermined once again through the curve fitting method based on all the experimental data with different load ratios.

3. Application of the extended McEvily model

3.1. Experimental data of aluminum alloy 6013 [22]

The experimental data of fatigue crack growth rate of aluminum alloy 6013 with load ratios ranging from −1.0 to 0.7 are shown in Fig. 1. However, the detailed test conditions cannot be given herein because the corresponding necessary information is not described in Ref. [22]. It can be seen from Fig. 1 that the experimental data are relation of fatigue crack growth rate, da/dN and the stress intensity factor range, ΔK. While in the extended McEvily model the independent variable is crack length, a. Then the following equation is employed to transform ΔK into a

$$\Delta K = Y(a)\Delta\sigma\sqrt{\pi a} \tag{15}$$

where $\Delta\sigma$ is the applied stress range. There is little test information about aluminum alloy 6013 in Ref. [22] except the experimental data of da/dN versus ΔK under different load ratios. So in this paper, the accurate $Y(a)$ cannot be given and for simplification, the geometrical factor, $Y(a)$ is assumed to be constant and is set to be 0.65 for an assumed semi-circular crack according to Ref. [9]. The above assumption may not be the best. However, the following discussions and analyses in the text are fully based on the transformed data, i.e. da/dN versus a. Then the value of $Y(a)$ will not have a significant effect on the conclusions drawn in this paper. Besides, the maximum stress, $\sigma_{\max}$, is set to be 150 MPa. The experimental data after transformation are illustrated in Fig. 2.

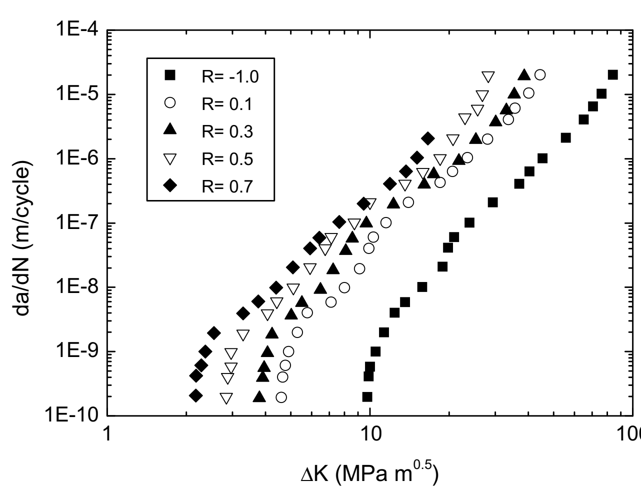

Fig. 1. Experimental data of fatigue crack growth rate of aluminum alloy 6013 [22].

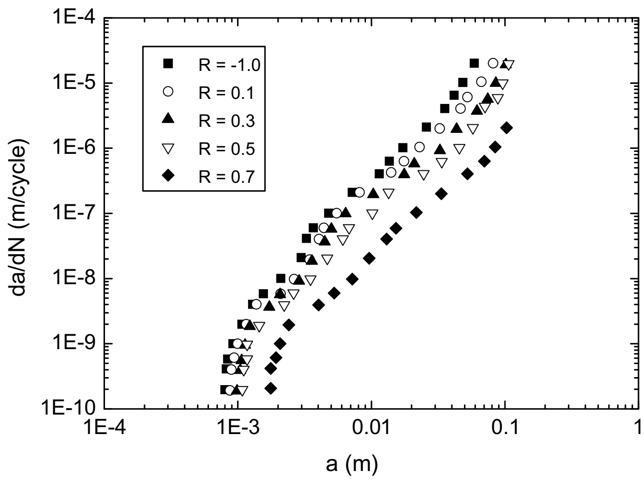

Fig. 2. Experimental data after being transformed.

3.2. Nonlinear least squares curve fitting

The parameters in the extended McEvily model for fatigue crack growth analysis can be determined through the nonlinear least squares curve fitting method based on the above experimental data. For each load ratio, a group of parameters can be estimated. The fitting parameters are listed in Table 1. The fitting results and the corresponding experimental data for different load ratios are shown in Fig. 3. It is pointed out in Ref. [22] that a significant closure is observed in terms of compliance measurements for the tests with load ratios $R < 0.7$, while at the highest load ratio, 0.7, the experimental data are closure free. Then in the present paper parameters k and $K_{\rm op,max}$ are equal to zero for the load ratio, 0.7. The curve fitting is mainly to obtain the values of $\Delta K_{\rm eff,th}$ and $K_{\rm op,max}$ under different load ratios. Though the extended McEvily model is physically based, the obtained parameters are the best fitting results and totally mathematically based when the curve fitting is carried out. Hence, scatter for the same parameters under different load ratios will always exist as shown in Table 1 and accordingly it is unsuitable to adopt one group of parameters under a certain load ratio as the material constants. In the following sections, a procedure will be suggested for best determining a group of parameters as the material constants.

3.3. Function of $K_{op,max}$ against load ratio, R

Based on the fitting results of parameter $K_{\rm op,max}$, the function of $K_{\rm op,max}$ against load ratio, R is suggested as follows

$$K_{\rm op,max} = \begin{cases} 2.70362 - 0.35529R & -1.0 \leqslant R \leqslant 0.5 \\ \dfrac{2.65}{1+e^{\frac{R-0.54642}{0.0165}}} & 0.5 < R < 1.0 \end{cases} \tag{16}$$

The fitting results and the predicted results using Eq. (16) are shown in Fig. 4. It is pointed out in Ref. [22] that for 6013 aluminum alloy crack closure is experimentally detected with load ratio $R < 0.7$, while at load ratio, 0.7, the experimental data are closure free. It is reported by Boyce and Ritchie [23] that for Ti-6Al-4V alloy no closure is detected at load ratio, $R > 0.5$ and in the corresponding experimental procedure load ratios range from 0.1 to 0.955. Donald and Paris [24] found that analysis of the load-displacement data reveals no evidence of non-linearity associated with crack closure for 6061-T6 and 2024-T3 aluminum alloys at load ratio of 0.7. It is important to note that for Ti-6Al-4V alloy crack closure was not observed at load ratio $R = 0.8$. However, significant levels of crack closure were detected at load ratio $R = 0.1$ [25]. Then it can be concluded that crack closure will disappear at load ratios higher than a certain value. Thus besides the data of load ratios from -1.0 to 0.7, the values of $K_{\rm op,max}$ for load ratios, $R = 0.8$, 0.9 and 0.95 are assumed to be zero for curve fitting as illustrated in Fig. 4. Eq. (16) is obtained by linear curve fitting for load ratios, $R \leqslant 0.5$ and sigmoid fitting for load ratios, $R \geqslant 0.3$, respectively and it is recommended that the sigmoid fitting results are only valid for load ratios above 0.5.

It can be seen from Fig. 4 that $K_{\rm op,max}$ will firstly decrease slightly before the load ratio, R, reaches 0.5 and then drop sharply to zero as the load ratio ranges from 0.5 to 0.7. Error analysis for $K_{\rm op,max}$ between the predicted results and the fitting results is made in Table 2. It is shown that the predicted results agree well with the corresponding fitting results.

3.4. Function of $\Delta K_{eff,th}$ against load ratio, R

According to the fitting results of parameter $\Delta K_{\rm eff,th}$, the function of $\Delta K_{\rm eff,th}$ against load ratio, R, is proposed as follows:

$$\Delta K_{\rm eff,th} = 1.48221 + \frac{0.24655}{4(R - 0.46288)^2 + 0.34596} \tag{17}$$

Table 1
Fitting parameters according to fatigue experimental data of aluminum alloy 6013 with different load ratios

R	A	m	n	k	K_c	r_e	σ_V	$K_{\rm op,max}$	$\Delta K_{\rm eff,th}$
	$\text{MPa}^{-m}\,\text{m}^{1-m/2}$	–	–	m^{-1}	$\text{MPa}\sqrt{\text{m}}$	m	MPa	$\text{MPa}\sqrt{\text{m}}$	$\text{MPa}\sqrt{\text{m}}$
−1.0	6.0068E−10	2.60	6.00	9346	48.35	2.18E−07	408	3.05	1.50
0.1	2.9948E−10	2.70	6.00	9345	57.03	1.00E−07	450	2.70	1.80
0.3	1.8894E−10	2.66	6.00	10000	62.45	1.00E−07	426	2.60	2.00
0.5	1.5274E−10	2.61	6.00	10000	61.67	1.10E−07	448	2.50	2.20
0.7	9.3434E−10	2.52	6.42	0.00	65.00	1.00E−06	385	0.00	1.90

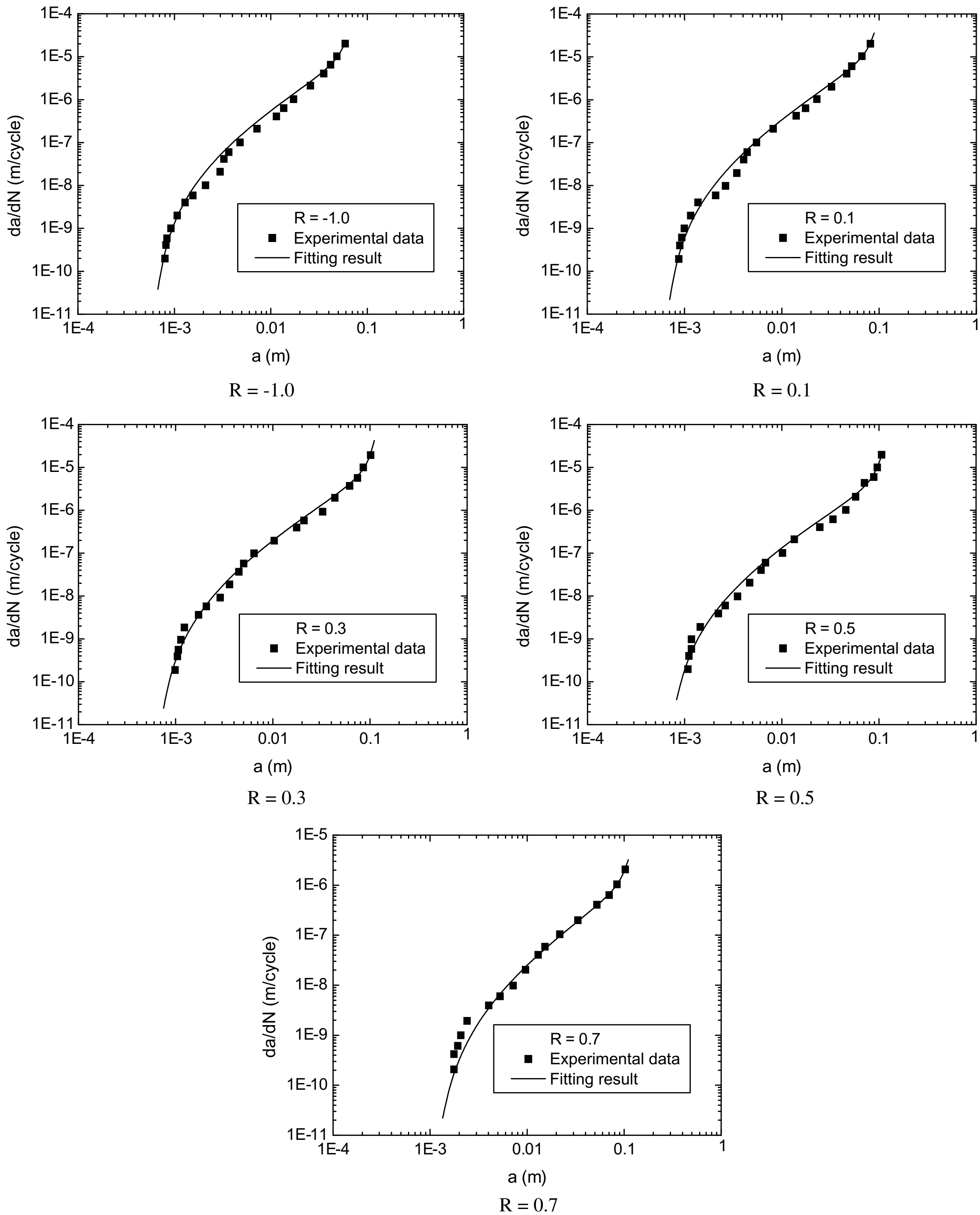

Fig. 3. Fitting results corresponding to fatigue experimental data with five load ratios ranging from −1.0 to 0.7.

The fitting results and the predicted results using Eq. (17) are illustrated in Fig. 5. Error analysis for $\Delta K_{\mathrm{eff,th}}$ between the predicted results and the fitting results is listed in Table 2. Results indicate that the predicted results are in reasonably good agreement with the corresponding fitting results.

In Ref. [18] the following equation was adopted to represent the relation between $\Delta K_{\mathrm{eff,th}}$ and load ratio, R

$$\Delta K_{\mathrm{eff,th}} = \Delta K_{\mathrm{eff,th0}}(1-R)^{\gamma} \tag{18}$$

where $\Delta K_{\mathrm{eff,th0}}$ is the corresponding value at zero load ratio and γ is the material constant. Eq. (18) reveals that $\Delta K_{\mathrm{eff,th}}$

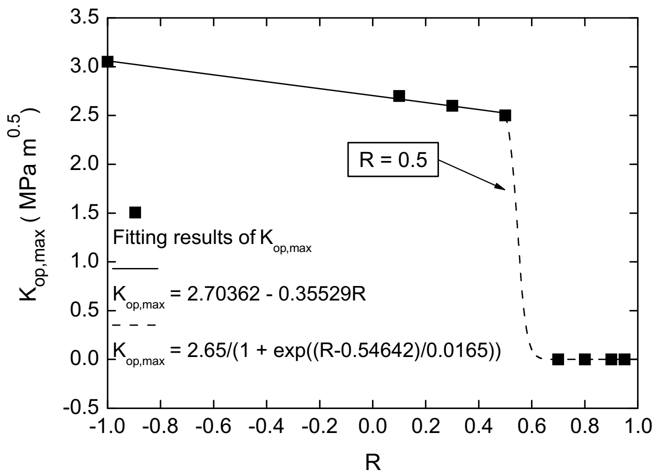

Fig. 4. Fitting results and predicted results of parameter $K_{op,max}$.

Table 2
Error analysis of $K_{op,max}$ and $\Delta K_{eff,th}$ between predicted results and fitting results

R	$K_{op,max}$ (MPa$\sqrt{m}$)			$\Delta K_{eff,th}$ (MPa$\sqrt{m}$)		
	Fitting results	Predicted results	Error (%)	Fitting results	Predicted results	Error (%)
−1.0	3.050	3.059	0.291	1.500	1.510	0.655
0.1	2.700	2.668	−1.196	1.800	1.765	−1.999
0.3	2.600	2.597	−0.114	2.000	2.028	1.360
0.5	2.500	2.526	1.028	2.200	2.184	−0.747
0.7	0.000	0.00024	–	1.900	1.914	0.737

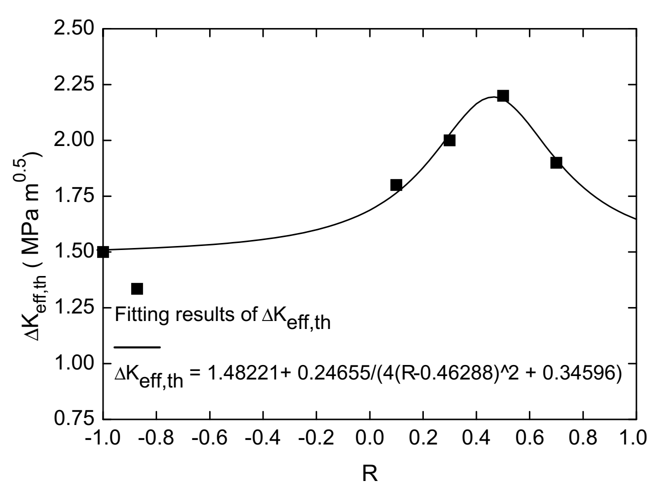

Fig. 5. Fitting results and predicted results of parameter $\Delta K_{eff,th}$.

will continually decrease as load ratio increases. However, the following discussions and analyses will show that the above equation is unsuitable to describe the relation between $\Delta K_{eff,th}$ and load ratio.

Assuming that both $\Delta K_{eff,th}$ and $K_{op,max}$ are constant and independent of load ratio, R, the following equation [23,26,27] has been proposed

$$\Delta K_{eff,th} = \begin{cases} K_{max,th} - K_{op,max} & R < R_c (K_{min,th} < K_{op,max}) \\ K_{max,th} - K_{min,th} = \Delta K_{th} & R > R_c (K_{min,th} > K_{op,max}) \end{cases} \quad (19)$$

where $K_{max,th}$ and $K_{min,th}$ are the maximum and minimum stress intensity factors at the threshold level, respectively; ΔK_{th} is the stress intensity factor range at the threshold level; R_c is the critical load ratio at which $K_{min,th} = K_{op,max}$. Under above conditions, $K_{max,th}$ is independent of load ratio, R below R_c and ΔK_{th} is also independent of load ratio, R above R_c. Plotted as ΔK_{th} versus $K_{max,th}$, the transition exhibits as a dramatic 'L' shape, as shown in Fig. 6. However, many experimental results indicate that the value of ΔK_{th} is not invariant at $R > R_c$ and ΔK_{th} decreases with increasing load ratio, R as shown in Fig. 2 in Ref. [23]. At the same time, the following equations are largely adopted to represent the effect of load ratio R on ΔK_{th} [28–30]

$$\Delta K_{th} = \Delta K_{th,R0} - B_1 R \quad (20)$$

$$\Delta K_{th} = \Delta K_{th,R0}(1 - R)^{\gamma} \quad (21)$$

where $\Delta K_{th,R0}$ is the threshold stress intensity factor range value corresponding to $R = 0$ and B_1 and γ are material constants. According to Schijve [31], γ is between 0.5 and 1.0. Both Eqs. (20) and (21) reveal that ΔK_{th} tends to decrease as load ratio increases. The experimental data [23] of ΔK_{th} and $K_{max,th}$ employed here with different load ratios ranging from 0.1 to 0.955 are shown in Fig. 7. It can be seen that the experimental results are distinctly different from the simple model suggested by Schmidt and Paris [27]. $K_{max,th}$ is not constant any more at $R < R_c$ and tends to increase as load ratio increases. ΔK_{th} is also not constant any more at $R > R_c$ and continues to decrease with increasing load ratio R. Especially, ΔK_{th} decreases approximately linearly with increasing K_{max} at $R > R_c$ where crack closure is

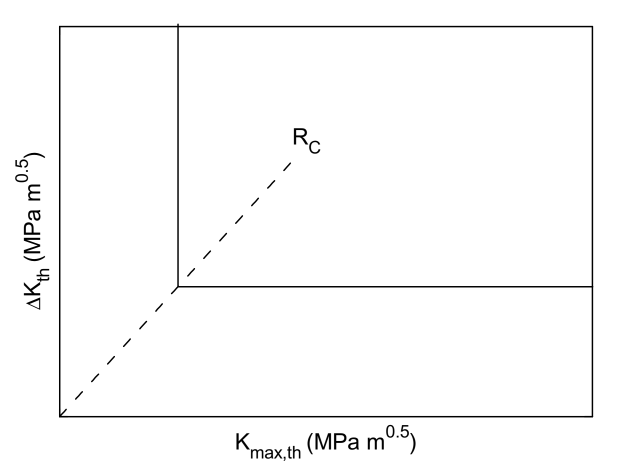

Fig. 6. Schematic illustration of ΔK_{th} versus $K_{max,th}$ proposed by Schmidt and Paris [23].

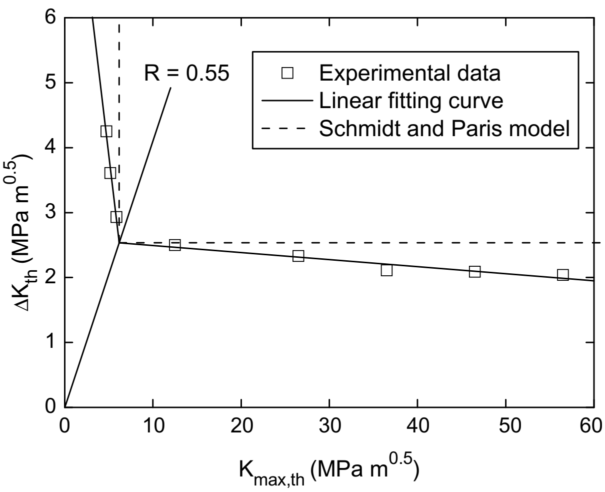

Fig. 7. Experimental data of ΔK_{th} and $K_{max,th}$ in Ref. [23].

not experimentally detected as shown in Fig. 7. In fact, it has been reported [23,32,33] that in the region where no crack closure occurs at $R > R_c$ the fatigue threshold ΔK_{th} is dominated by K_{max} and the function of ΔK_{th} versus K_{max} has been proposed as follows

$$\Delta K_{th} = \Delta K_{th,K_{max}0} + B_2 K_{max} \qquad R > R_c \tag{22}$$

$$\Delta K_{th} = \alpha (K_{max})^{\beta} \qquad R > R_c \tag{23}$$

where $\Delta K_{th,K_{max}0}$ is the threshold stress intensity factor range value corresponding to $K_{max} = 0$. B_2 is the slope of the decrease in threshold with increasing K_{max}. α and β are material constants. However, for simplification, the conventional explanation of load ratio effects on the fatigue threshold at $R > R_c$ is employed in the present paper, i.e. ΔK_{th} is still related to load ratio, R. It is reported in Ref. [23] that at low load ratios, $R < 0.5$, $K_{op,max}$ values were found to be approximately constant. Then $\Delta K_{eff,th}$ has to increase as $K_{max,th}$ increases according to Eq. (19). Nevertheless, $K_{op,max}$ decreases slightly with increasing load ratio, R as shown in Fig. 4. So, as discussed above, $\Delta K_{eff,th}$ will firstly increase as load ratio increases at $R < R_c$ and then decrease at $R > R_c$.

On the other hand, Donald and Paris [24] has observed a lack of correlation of fatigue crack growth rate using the traditional definition of ΔK_{eff} which is given by Eq. (2). It is found in plots of da/dN against ΔK_{eff} that significant scatter exists only in the near threshold region and ΔK and ΔK_{eff} data at the threshold level exhibit a fully reverse order [34], i.e. ΔK_{th} decreases and ΔK_{eff} increases, respectively as load ratio increases. Kujawski [34] has pointed out that the effect of crack closure on crack driving force given by Eq. (2) might be greatly exaggerated. Chen et al. [35] found in tests that the contribution of the cyclic loading portion below the opening load to the fatigue crack growth should be taken into account. It is thus suggested that the conventional definition of the effective driving force, ΔK_{eff} should be modified to take the contribution into consideration. This certainly results in a larger effective driving force compared with the conventional crack closure evaluation. Paris et al. [22] have proposed a partial crack closure model exhibiting 'Donald's effect' which can be expressed as follows:

$$\Delta K_{eff} = K_{max} - \frac{2}{\pi} K_{op} \tag{24}$$

Results indicate that the value of $\Delta K_{eff,th}$ would still tend to increase in a certain degree with increasing load ratio though the previous partial crack closure model is adopted.

Based on the above discussions, the conclusion can be drawn that $\Delta K_{eff,th}$ exhibits increase at $R < R_c$ where crack closure exists and decrease at $R > R_c$ where the experimental data are closure free. Thus the present result appeared in Fig. 5 is reasonable.

3.5. Determination of parameters in the extended McEvily model

According to Eqs. (10)–(14), it is impossible to obtain all the parameters in the extended McEvily model through the nonlinear least squares curve fitting method with all the experimental data of different load ratios. However, the parameters A and m can be determined based on all the experimental data. The following equation can be derived from Eq. (10)

$$\frac{da}{dN}\left[1 - \left(\frac{K_{max}}{K_c}\right)^n\right] = AM^m \tag{25}$$

In this paper, parameters n, k, K_c, r_e and σ_V are determined through calculating the average values of the corresponding data under different load ratios. Then, by substituting $K_{op,max}$ (R) and $\Delta K_{eff,th}$ (R) relations which are described by Eqs. (16) and (17), respectively and the final values of parameters n, k, K_c, r_e and σ_V into the extended McEvily model, the values of M and $\frac{da}{dN}\left[1 - \left(\frac{K_{max}}{K_c}\right)^n\right]$ under different load ratios can be obtained. Fig. 8 shows all the data with

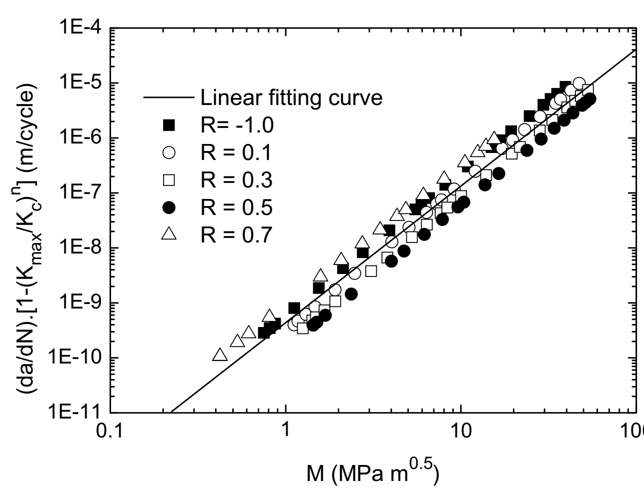

Fig. 8. Linear curve fitting for determining parameters A and m using all experimental data.

Table 3
Parameter values employed as the material constants

Parameters	A	m	n	k	K_c	r_e	σ_V
	$MPa^{-m}\,m^{1-m/2}$	–	–	m^{-1}	$MPa\sqrt{m}$	m	MPa
Values	4.3601E−10	2.487	6.08	9673	58.90	3.05E−07	423

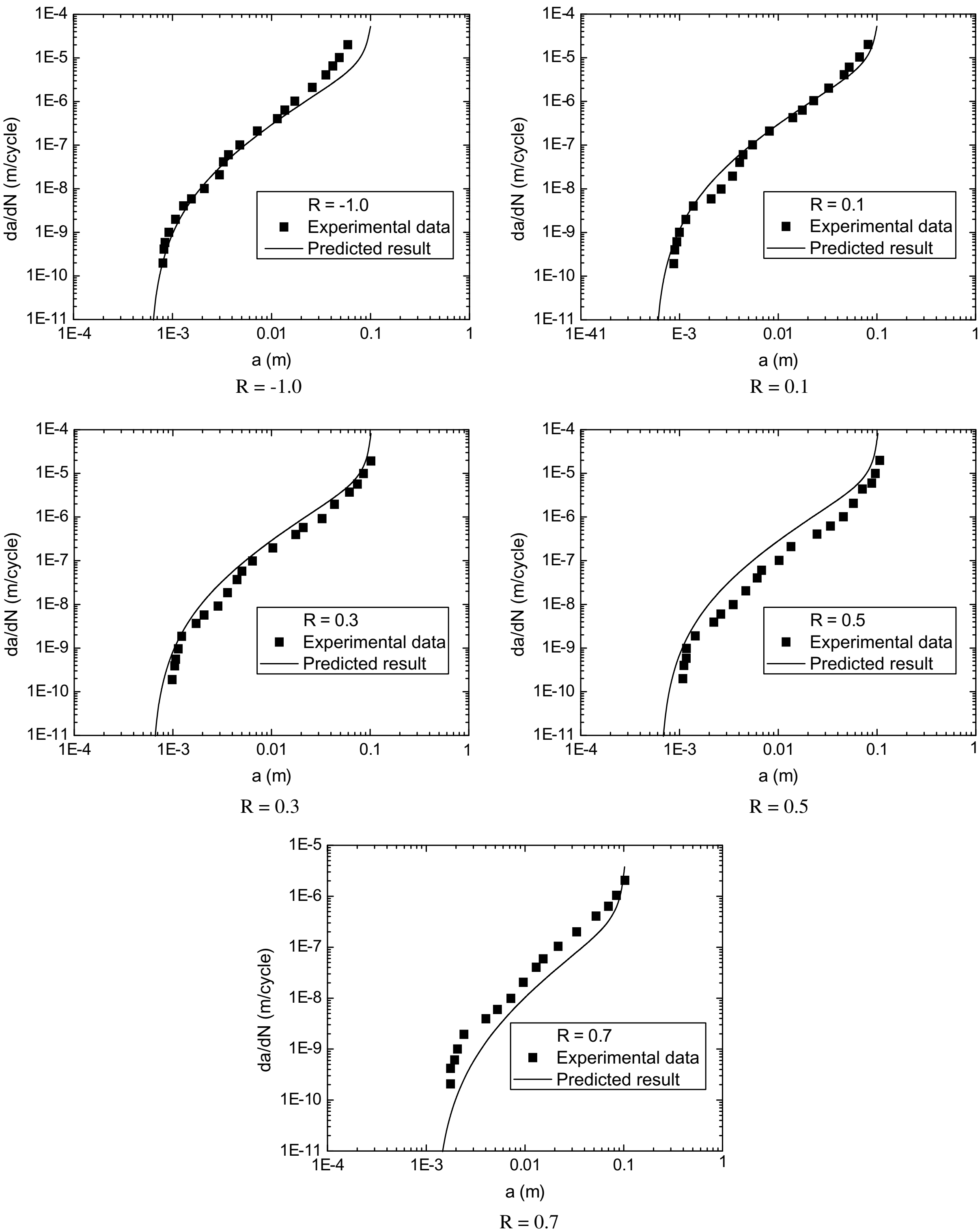

Fig. 9. Comparison between the predicted results derived from the same parameters for different load ratios and the corresponding experimental data.

different load ratios in plot of $\frac{da}{dN}\left[1-\left(\frac{K_{max}}{K_c}\right)^n\right]$ versus M. It can be seen that the data with different load ratios collapse into a narrow band. The linear fitting curve is described as follows:

$$\log_{10}^{\frac{da}{dN}\left[1-\left(\frac{K_{max}}{K_c}\right)^n\right]} = -9.36053 + 2.4874\log_{10}^{M} \tag{26}$$

The parameters A and m can be determined according to Eq. (26). Finally, the following results listed in Table 3 are adopted as the material constants.

Based on the same parameters, the predicted results of fatigue crack growth rate are plotted in Fig. 9 with the corresponding experimental data with different load ratios. It can be seen that the predicted results are in good agreement with the experimental data in general. However, compared with the corresponding experimental data, the predicted result for load ratio $R = 0.7$ is not very good. It can be seen that relatively larger errors exist for parameters A, m, n, k, K_c, r_e and σ_V between the fitting result at $R = 0.7$ and the final determined parameters from Tables 1 and 3. This may arise from the approach for determining the material constants especially for parameters n, k, K_c, r_e and σ_V. If some of the parameters such as k, K_c, r_e and σ_V can be obtained in advance, the error might be reduced. Besides, the predicted result may be improved if the piecewise linear curve fitting method is employed. However, the extended McEvily model is physically based and not a purely curve fitting. So, it can be concluded that the extended McEvily model presented in this paper can account for the load ratio effect reasonably well. Based on our previous work [18], this model can also be expected to be able to explain other fatigue phenomena.

In this paper, all the parameters A, m, n, k, K_c, r_e and σ_V are assumed to be unknown ahead of time though they are regarded as material constants in the extended McEvily model and are determined through the curve fitting method based on the experimental data. This condition is the most complicated one. In fact, parameters K_c, r_e and σ_V can be obtained according to the corresponding information in the quasi-static condition and parameter k can also be fixed based on our previous work for macroscopic crack growth. Then apart from $\Delta K_{eff,th}$ and $K_{op,max}$ which are regarded as variables under different load ratios, parameters A, m and n are only left to be determined. In this doing, the extended McEvily model is convenient for use in practice.

4. Summary and conclusions

The McEvily model cannot only account for the effects of initial crack size and load sequence, but also explain various other phenomena of metal fatigue observed in tests. The model has been successfully applied to many fatigue problems and shows great promise. Based on the previous work of our group, further extension to the McEvily model for fatigue crack growth analysis is proposed in this paper mainly accounting for the following two aspects: (1) to introduce parameter m to replace the fixed slope of 2 in McEvily model; (2) to regard $K_{op,max}$ and $\Delta K_{eff,th}$ as functions of load ratio, R and to determine them through the nonlinear least squares curve fitting method.

Based on the fitting results, functions of $K_{op,max}$ and $\Delta K_{eff,th}$ versus load ratio, R are proposed, respectively. It should be noted that the functions may be different for different materials. Results indicate that the value of $K_{op,max}$ tends to decrease slightly as the load ratio increases where crack closure is experimentally detected. The conclusion drawn for parameter $\Delta K_{eff,th}$ from the present fitting results and discussions on the experimental data in the published literature shows that $\Delta K_{eff,th}$ exhibits increase at $R < R_c$ where crack closure exists and decrease at $R > R_c$ where the experimental data are closure free.

In this extended McEvily model, parameters A, m, n, k, K_c, r_e and σ_V are regarded as material constants. However, these parameters are assumed to be unknown in advance and they are determined through the curve fitting method based on the experimental data. The curve fitting is mainly to obtain the values of $\Delta K_{eff,th}$ and $K_{op,max}$ under different load ratios. Though the extended McEvily model is physically based, the obtained parameters are the best fitting results and totally mathematically based when the curve fitting is carried out. Therefore, scatter for the same parameters under different load ratios will always exist and accordingly it is unsuitable to adopt one group of parameters under a certain load ratio as the material constants. The method to determine the group of parameters employed as the material constants is suggested in this paper. Comparison between the predicted results and the corresponding experimental data with different load ratios reveals that the extended McEvily model can account for the load ratio effect reasonably well. This also implies that the method put forward to estimate the material constants is valid. On the other hand, in fact, parameters k, K_c, r_e and σ_V can be estimated in advance especially for macroscopic crack growth. Then besides $\Delta K_{eff,th}$ and $K_{op,max}$ which are viewed as variables under different load ratios, parameters A, m and n are only left to be determined. Therefore, the extended McEvily model is convenient for use in practice.

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