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Effect of corrosion models on the time-dependent reliability of steel plated elements

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Abstract

Time-dependent reliability considering corrosion and fatigue has received increasing attention recently. Many corrosion models have been proposed. In this paper, a new corrosion model which could better describe the corrosion process of actual steel structures under corrosive environment is proposed. This model is also compared with other existing corrosion models. The effect of corrosion models on the time-dependent reliability is studied using a steel plated element which has found wide applications in engineering structures. The advantages and the flexibility of the present corrosion model are demonstrated.

Keywords: Time-dependent reliability; Corrosion model; Corrosion mechanism; Steel plated element

1. Introduction

Marine environments are recognized to be very corrosive for mild and low alloy steels. For reasons of economy, such steels are still the preferred materials for many engineering structures such as ship hulls and offshore structures. Statistics for ship hulls show that around 90% of ship failures are attributed to corrosion, including corrosion fatigue [1–3]. For oil tankers and bulk carriers there have been a number of sinkings and environmental disasters attributed to poorly maintained and highly corroded hulls [4–6].

Paint coatings and cathodic protection are the main means employed to protect steel against corrosion. Provided maintenance is adequate and the corrosion

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protection system (CPS) is properly applied, there should be little concern about corrosion. However, field observation reveals that maintenance procedures are not always sufficient, especially for aging ships where the CPS may be ineffective or break down. Also, there are some areas of ships such as the lower parts of the holds in bulk coal and iron ore carriers [7], which are not or cannot be protected. Therefore corrosion remains one of the dominant factors which led to ship structural failures.

Due to the casualties of aging vessels during the last decade, the safety assessment of hull structures subjected to corrosion and fatigue has been of increasing interest (e.g. [8–17]). Reliability considering corrosion and fatigue is often called time-dependent reliability [18] because both corrosion and fatigue is a function of time. In the assessment of reliability under corrosion which is the main concern of the present paper, one of the key factors which influences the result is the choice of the corrosion model.

A purely theoretical model of the likely loss of material due to corrosion based on the actual corrosion mechanism is extremely difficult due to the complexity of the problem [19–23]. Many factors including the CPS and various operational parameters will influence the corrosion rate. Therefore, most of the corrosion models used in the literature are based on the assumption or actual measurement. Due to the differences in the size of databases and the quality of data, the corrosion models are different. This could have significant impact on the assessed time-dependent reliability. In this paper, various existing corrosion models are studied and based on the understanding of the corrosion mechanism, a new corrosion model which could better describe the corrosion process of actual steel structures under corrosive environment is proposed. This model is also compared with other existing corrosion models. Next, the effect of corrosion models on the time-dependent reliability is studied using a steel plated element which has found wide applications in engineering structures. The advantages and the flexibility of the present corrosion model are demonstrated.

2. Corrosion mechanism and corrosion modeling

Depending on the exposure environment, marine corrosion may be divided into four categories [22]: (1) immersion; (2) splash/tidal zone; (3) atmospheric; and (4) semi-enclosed space. In this paper only the immersion corrosion of mild and low alloy steels under marine conditions (such as at sea) is considered. Ships are a typical example of marine immersion corrosion.

Ship structures operate in a complex environment. Water properties such as salinity, temperature, oxygen content, pH level and chemical composition can vary according to location and water depth. Also the inside face of plates will be exposed to aggressive environments existing in cargo tanks. The structures are often protected, either with paints or with cathodic systems that deliver a current intensity to the protected metal surface inhibiting the corrosion process. Therefore, the corrosion rate of ship hull structures is influenced by many factors including the CPS (e.g. coating and anodes) and various operational parameters such as the percentage of time in blast, the type of cargo, component location and orientation, level of oxygen, temperature, degree of flexibility, frequency and method of tank cleaning, maintenance and repair. A summary of the environmental factors which are considered to have possible effects on corrosion of mild and low alloy steels is given in Table 1 [23].

Two main corrosion mechanisms are generally present in steel plates. One is a general wastage that is reflected in a generalized decrease of plate thickness. Another mechanism is pitting which consists of much localized corrosion with very deep holes appearing in the plate. In fact, pitting can lead to leakage but in general, because it is much localized, it does not affect the mean in-plane stress distribution in plate.

Table 1
Environmental factors in marine corrosion [23]

Factor	Effect on initial corrosion rate	Effect on steady state corrosion rate	Influenced by
Biological			
Bacterial	None	Reduces and probably controls rate	Temperature of Seawater
Biomass/plant life			NaCl concentration
Animal life			Water velocity
			Suspended solids
			Pollutant type and level
			Percentage wetting
Chemical			
O_2	Directly proportional	None, if corrosion controlled by O_2 transfer rate	Seawater temperature
			NaCl
CO_2	Little effect	Little effect	
NaCl	Inversely proportional	Proportional	Unimportant in open oceans
			Fresh water inflows
			Effect of biological activity
pH	Little effect	Little effect	
Carbonate solubility	Little effect	Little effect	
Pollutants	Varies	Varies	Geographical location
Physical			
Temperature	Directly proportional	Proportional	Geographical location
Pressure			Not significant for shallow waters
Water velocity	Little effect	Little effect	Geographical location
Suspended solids		Little effect, if any	Geographical location
Percentage wetting	Proportional for tidal and splash zones	Proportional for tidal and splash zones	Location, weather patterns

Therefore, pitting is not accounted for in this paper and general corrosion is modeled as a monotonic decrease in plate thickness.

Because corrosion is a function of many variables, many of an uncertain nature, a probabilistic model is more appropriate to describe the expected corrosion. Melchers [23] constructed a probabilistic phenomenological model from a mean value expression and an expression picking up random and other uncertainties not modeled in the mean value expression, as follows:

$$C(t, P, E) = f_n(t, P, E) + \varepsilon(t, P, E), \tag{1}$$

where $C(t, P, E)$ is the weight loss of material, $f_n(t, P, E)$ is a mean value function, $\varepsilon(t, P, E)$ is a zero mean error function, t is time, P is a vector of the parameters which define the CPS and E is a vector of the environmental conditions.

A fundamental study on the corrosion mechanism of the unprotected steel specimen has been carried out by Melchers and his colleagues [20–23]. Some understanding on the immersion corrosion mechanism has been achieved. For unprotected steel structures, the corrosion process can be divided into four stages (see Fig. 1):

(1) initial corrosion;
(2) oxygen diffusion controlled by corrosion products and micro-organic growth;
(3) limitation on food supply for aerobic activity; and
(4) anaerobic activity.

For some stages, the main environmental parameters E have been recognized and quantified but for other stages, better understanding of the corrosion mechanism is still required. Table 1 also indicated this current state-of-the-art where many question marks exist. Therefore, further research is needed in order to apply their probabilistic phenomenological model.

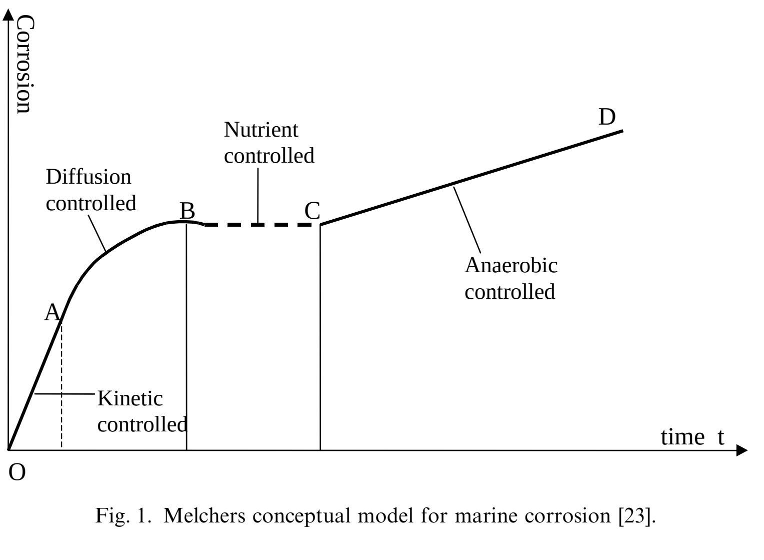

Fig. 1. Melchers conceptual model for marine corrosion [23].

3. Existing corrosion models

In most of the studies on time-dependent reliability of ship structures (e.g. [8–13,15–17]), the effect of corrosion was represented by an uncertain but constant corrosion rate, which resulted in a linear decrease of plate thickness with time. However, experimental evidence of corrosion reported by various authors shows that a nonlinear model is more appropriate. Southwell et al. [24] proposed a linear and a bilinear model. By interpreting their original model parameters as a mean value and through statistical analysis, these two models have also been extended by Melchers [23] to give the second statistical moment. Furthermore, an alternative power expression is also given. The extended Southwell's models are:

Extended Southwell linear model:

$$\begin{aligned} \mu_d(t) &= 0.076 + 0.038t, \\ \sigma_d(t) &= 0.051 + 0.025t. \end{aligned} \tag{2}$$

Extended Southwell bilinear model:

$$\begin{aligned} \mu_d(t) &= \begin{cases} 0.09t, & 0 < t < 1.46 \text{ years}, \\ 0.076 + 0.038t, & 1.46 < t < 16 \text{ years}, \end{cases} \\ \sigma_d(t) &= \begin{cases} 0.062t, & 0 < t < 1.46 \text{ years}, \\ 0.035 + 0.017t, & 1.46 < t < 16 \text{ years}. \end{cases} \end{aligned} \tag{3}$$

Melchers–Southwell nonlinear model:

$$\begin{aligned} \mu_d(t) &= 0.084t^{0.823}, \\ \sigma_d(t) &= 0.056t^{0.823}. \end{aligned} \tag{4}$$

Melchers [22] also suggested a trilinear and another power approximation for corrosion wastage thickness, which are given as

Melchers trilinear model:

$$d(t) = \begin{cases} 0.170t, & 0 \leqslant t < 1, \\ 0.152 + 0.0186t, & 1 \leqslant t < 8, \\ -0.364 + 0.083t, & 8 \leqslant t \leqslant 16. \end{cases} \tag{5}$$

Melchers power model:

$$d(t) = 0.1207t^{0.6257}, \tag{6}$$

where $d(t)$ is the thickness of the corrosion wastage at time t in the deterministic sense, $\mu(t)$ and $\sigma(t)$ are the mean and standard deviation of the thickness of the corrosion wastage at time t in the probabilistic sense.

Based on some observations reported in the literature, Guedes Soares and Garbatov [14] proposed a nonlinear model to describe the growth of corrosion. They divided the whole corrosion process into three phases. In the first phase, it is assumed that there is no corrosion because the CPS is effective. The first stage, $t \in [O, A]$ in Fig. 2, depends on many factors and statistics show that in ships it varies in the range

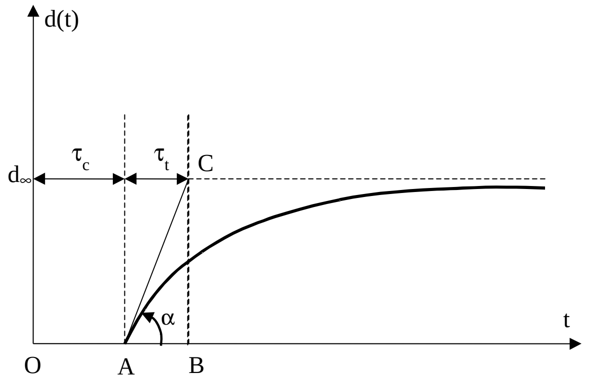

Fig. 2. Thickness of corrosion wastage as a function of time [14].

of 1.5–5.5 years [3] or in the range of 5–10 years [25]. The second phase is initiated when the corrosion protection is damaged and corresponds really to the existence of corrosion, which decreases the thickness of plate, $t \in [A, B]$ in Fig. 2. This process may last a period around 4–5 years in ship plating [14]. The third phase corresponds to a stop in the corrosion process and the corrosion rate becomes zero, $t \in [B, \infty]$ in Fig. 2. Corroded material stays on the plate surface, protecting it from the contact with the corrosive environment and the corrosion process stops. Cleaning the surface or any involuntary action that removes that surface material originates the new start of the nonlinear corrosion growth process. This removal is not considered in their study [14] and the present study.

The model proposed by Guedes Soares and Garbatov [14] was derived from the solution of a differential equation of the corrosion wastage

$$\tau_t r(t) + d(t) = d_\infty, \tag{7}$$

where d_∞ is the long-term thickness of the corrosion wastage, $d(t)$ is the thickness of the wastage at time t, and $r(t)$ is the corrosion rate and τ_t is the transition time, which may be calculated as

$$\tau_t = \frac{d_\infty}{tg\,\alpha}, \tag{8}$$

where α is the angle defined by AC and AB in Fig. 2.

The solution to Eq. (7) is

$$d(t) = \begin{cases} 0, & t \leqslant \tau_c, \\ d_\infty(1 - \mathrm{e}^{-(t-\tau_c)/\tau_t}), & t > \tau_c, \end{cases} \tag{9}$$

where τ_c is the coating life, which is equal to the time interval between the painting of the surface and the time when its effectiveness is lost.

In this model, three parameters τ_c, τ_t and d_∞ are used to describe the corrosion process. As an example, they used $d_\infty = 5\,\mathrm{mm}$, $\tau_t = 15.2$ year for an uncoated plate [14].

The corrosion model proposed by Paik et al. [26] is divided into two parts. One is related to the life of coating and the other is related to the progress of corrosion. They assumed that the corrosion starts immediately after coating effectiveness is lost which is similar to Guedes Soares and Garbatov [14].

The life of a coating essentially corresponds to the time when the corrosion starts after the new-building of vessels. The life of coatings may be assumed to follow the normal distribution, given by

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma_{\mathrm{cl}}} \exp\left\{ -\frac{(t-\mu_{\mathrm{cl}})^2}{2\sigma_{\mathrm{cl}}^2} \right\}, \tag{10}$$

where μ_{cl} is the mean value of coating life, σ_{cl} is standard deviation of coating life. In their paper [26], the result of Loseth et al. [25] is used. That is, the mean value of coating life is taken to be 5–10 years. A 5 years coating life may be considered to represent an undesirable situation, while 10 years would be representative of a relatively more desirable state of affairs. Also, according to Emi et al. [3], the coefficient of variation (COV) of coating life is about 0.4.

The wear of plate thickness due to corrosion may be generally expressed as a function of the time (year) after the corrosion starts, namely

$$d(t) = c_1(t - T_{\mathrm{cl}})^{c_2}, \tag{11}$$

where d is the wear of plate thickness due to corrosion; t is the elapsed time after the plate is used; T_{cl} is life of coating; c_1, c_2 is coefficients. The coefficient c_2 may be usually assumed to be 1/3 or pessimistically assumed to be 1, while the coefficient c_1 is indicative of the annual corrosion rate. Based on the probabilistic model proposed by Yamamoto et al. [27–29], Paik et al. [26] assumed that the probability density function of the corrosion rate follows the Weibull distribution. Hence, the cumulative distribution function and the probability density function of the coefficient c_1 are given by

$$F_{c_1}(x) = 1 - \exp\left[-\left(\frac{x}{w}\right)^k \right], \tag{12}$$

$$f_{c_1}(x) = \frac{k}{w}\left(\frac{x}{w}\right)^{k-1} \exp\left[-\left(\frac{x}{w}\right)^k \right], \tag{13}$$

where w is the unknown scale parameter, k is unknown shape parameter. By using the least-squares method, the unknown parameters w and k can be determined from the corrosion data collected. Once scale and shape parameters w and k are obtained, the mean and standard deviation of the coefficient c_1 can be calculated in terms of the Gamma function as follows:

$$\mu_{c_1} = \int_0^\infty x f_{c_1}(x)\,\mathrm{d}x = w\Gamma\left(1 + \frac{1}{k}\right), \tag{14}$$

$$\sigma_{c_1}^2 = \int_0^\infty (x - \mu_{c_1})^2 f_{c_1}(x)\,\mathrm{d}x = w^2\left[\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{1}{k}\right) \right]. \tag{15}$$

Applying the corrosion data for each primary member type, the probabilistic parameters, i.e. mean and standard deviation of the corrosion rate for primary members of bulk carriers are given in Paik et al. [26].

4. A new corrosion model

In Southwell's models [24] and Melchers' models [20–23], the CPS was not considered while in Guedes Soares' model [14] and Paik's model [26], the CPS was considered. However, in these models, the corrosion was assumed to start immediately after CPS effectiveness is completely lost or more accurately they define the instant as CPS life when corrosion starts. No interaction between the CPS and the environment was considered. In reality, the CPS such as coating will deteriorate gradually and the corrosion may start as pitting corrosions before the CPS loses its complete effectiveness [28]. If one defines a parameter q as the degree of effectiveness of the CPS, when the CPS is new and fully in function, q is equal to 1; when the CPS completely loses its effectiveness, q is equal to 0. This time should be defined as the life of the CPS. Therefore, for each CPS, two parameters T_{st} and T_{cl} may be used to describe its corrosion protection function. T_{st} is the instant at which the pitting corrosion starts. This quantity can be measured. T_{cl} is the life of the CPS at which the general corrosion starts. If we assume that the degree of effectiveness of the CPS is a measurable quantity, then T_{cl} can also be measured if we apply the CPS to an incorrosive material such as Titanium. Due to the fact that many factors such as location, environmental condition and stress level will affect the life of CPS, both T_{st} and T_{cl} may best be modeled as random variables. By distinguishing the corrosion initiation life T_{st} and the life of the CPS T_{cl}, one can see clearly that in the stage of pitting corrosion progress, both the CPS and the environmental parameters (macro and micro) will affect the corrosion rate. The corrosion rate can be defined by equating the volume of pitting corrosion to uniform corrosion. This can be regarded as the transition period and the corrosion rate increases. Therefore, this will also be called the corrosion acceleration period. After the CPS loses its complete effectiveness, general corrosion starts and the corrosion rate decreases due to the increasing thickness of the corrosion product (and the microbial biomass).

Therefore, the whole corrosion process can be divided into three stages: (1) no corrosion when the CPS is fully effective, $t \in [0, T_{st}]$; (2) corrosion accelerating when the pitting corrosion generates and progresses, $t \in [T_{st}, T_A]$; (3) corrosion decelerating, $t \in [T_A, T_L]$, where T_L is the life of the structure or the time at which repair and maintenance action takes place. In practice, the corrosion accelerating life T_A may be different from the CPS life, T_{cl} and at T_{cl} there may have some change in the corrosion rate. However, for the purpose of simplicity and ease of application, it is assumed that $T_A = T_{cl}$. The schematic representation of the new corrosion model is shown in Fig. 3.

For this shape of corrosion rate, a Weibull function is recommended to describe the corrosion rate and it is represented by

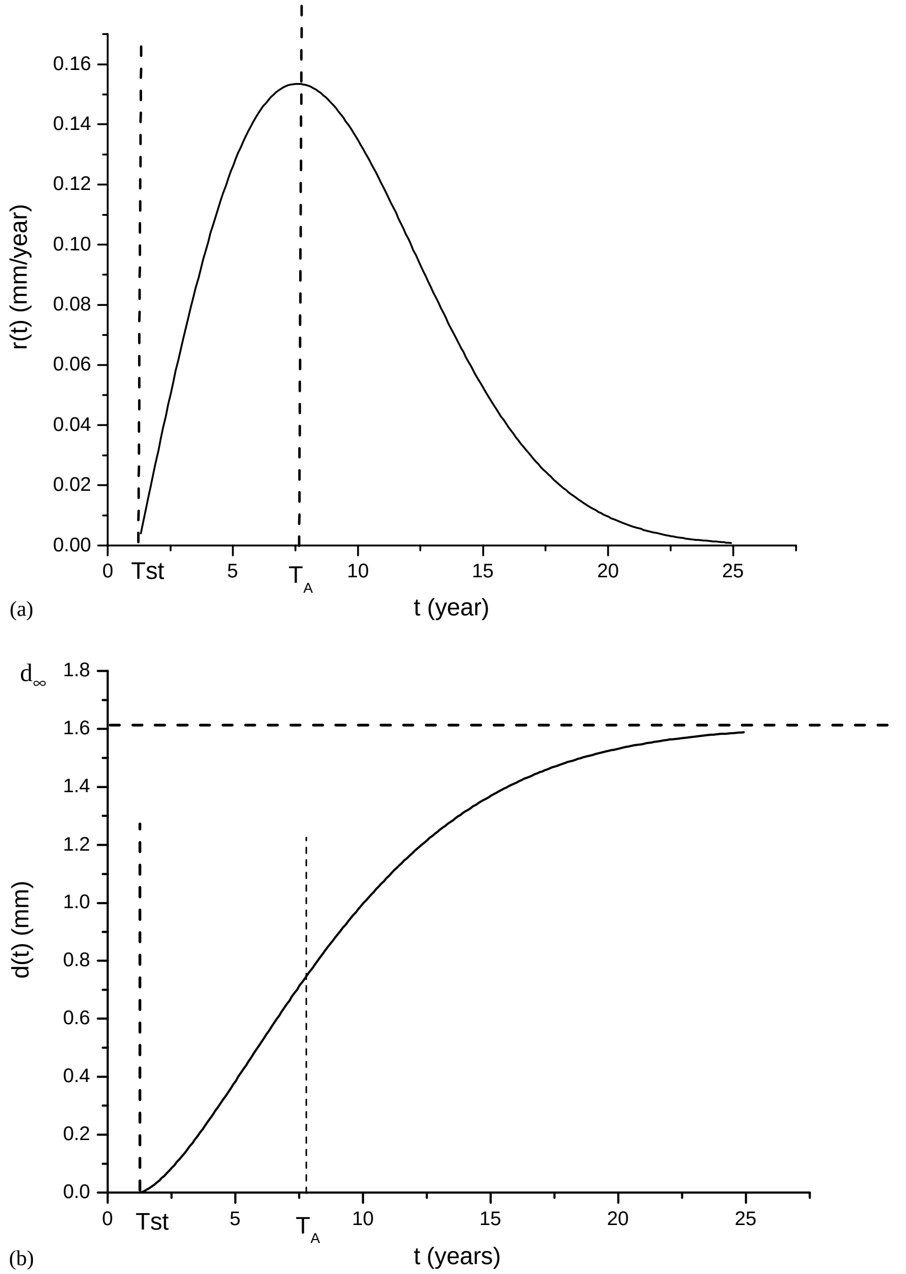

Fig. 3. Schematic representation of the new corrosion model. (a) Corrosion rate. (b) Corrosion wear.

$$r(t) = \begin{cases} 0, & 0 \leqslant t \leqslant T_{\mathrm{st}}, \\ d_\infty \dfrac{\beta}{\eta}\left(\dfrac{t - T_{\mathrm{st}}}{\eta}\right)^{\beta-1} \exp\left\{-\left(\dfrac{t - T_{\mathrm{st}}}{\eta}\right)^{\beta}\right\}, & T_{\mathrm{st}} \leqslant t \leqslant T_L, \end{cases} \tag{16}$$

where d_∞, β, η, T_{st} are four model parameters to be determined. The maximum corrosion rate will be achieved at the instant

$$T_A = \begin{cases} T_{\mathrm{cl}} = T_{\mathrm{st}} + \eta\left(\dfrac{\beta - 1}{\beta}\right)^{1/\beta}, & \beta > 1, \\ T_{\mathrm{st}}, & \beta \leqslant 1 \end{cases} \tag{17}$$

and the value is

$$r_{\max} = \begin{cases} d_\infty \dfrac{\beta}{\eta}\left(\dfrac{\beta - 1}{\beta}\right)^{(\beta-1)/\beta} \exp\left(\dfrac{\beta - 1}{\beta}\right), & \beta > 1, \\ d_\infty \beta/\eta, & \beta = 1, \\ \to \infty, & \beta < 1. \end{cases} \tag{18}$$

The instants at which the corrosion rate reaches the maximum under different conditions are shown in Fig. 4.

Using this corrosion model, the wear of thickness due to corrosion can be calculated by definition

$$d(t) = \begin{cases} 0, & 0 \leqslant t \leqslant T_{\mathrm{st}}, \\ d_\infty \left\{1 - \exp\left[-\left(\dfrac{t - T_{\mathrm{st}}}{\eta}\right)^{\beta}\right]\right\}, & T_{\mathrm{st}} \leqslant t \leqslant T_L. \end{cases} \tag{19}$$

The proposed model is flexible and can be fitted to most of the situations. Once the four parameters d_∞, β, η, T_{st} are known, the complete corrosion model is defined.

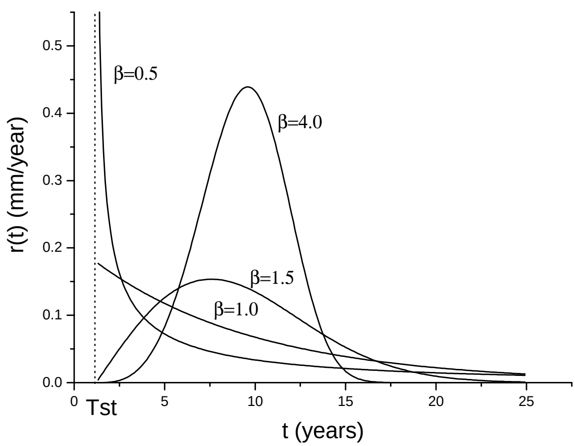

Fig. 4. The flexibility of the new corrosion model.

One advantage of the model is that it can describe different models of corrosion rate using the same format. Most of the existing corrosion models can be regarded as specific cases of the new corrosion model. These are discussed as follows.

When $\beta = 1$, Eq. (19) can be rewritten as

$$d(t) = d_\infty \left\{ 1 - \exp\left[-\left(\frac{t - T_{\rm st}}{\eta} \right) \right] \right\}. \tag{20}$$

This is the corrosion model proposed by Guedes Soares and Garbatov [14], i.e. Eq. (8).

When $\eta = 1$, if one applies the Taylor series expansion to Eq. (19) and only keeps the linear term, one can obtain

$$d(t) = d_\infty \left(\frac{t - T_{\rm st}}{\eta} \right)^\beta = d_\infty (t - T_{\rm st})^\beta. \tag{21}$$

This is the corrosion model proposed by Paik et al. [26], i.e. Eq. (11).

When $d_\infty = 0.1207$, $T_{\rm st} = 0$, $\eta = 1$, $\beta = 0.6257$, then Paik's model becomes

$$d(t) = 0.1207 t^{0.6257}. \tag{22}$$

This is the corrosion model proposed by Melchers [23], i.e. Eq. (6).

5. Determination of the parameters for the new corrosion model

In Eq. (19), there are four parameters to be determined. This is basically a nonlinear regression problem. In this section, we propose two methods to determine these parameters. In the first method, the four parameters are assumed to be deterministic while in the second method the four parameters are assumed to be random.

5.1. Method to determine four deterministic model parameters

When $T_{\rm st} \leqslant t \leqslant T_L$, one can rewrite Eq. (19) as

$$-\ln\left(-\ln\left(1 - \frac{d(t)}{d_\infty} \right) \right) = \beta \ln \eta - \beta \ln(t - T_{\rm st}). \tag{23}$$

Let us define

$$Y = -\ln\left(-\ln\left(1 - \frac{d(t)}{d_\infty} \right) \right), \quad X = \ln(t - T_{\rm st}),$$
$$A = \beta \ln \eta, \quad B = -\beta, \tag{24}$$

then

$$Y = A + BX. \tag{25}$$

So the relationship between A and B is linear. If the values of d_∞ and $T_{\rm st}$ are known, we can use the least squares method to determine the values of A and B:

$$B = L_{xy}/L_{xx}, \quad A = \bar{Y} - B\bar{X}. \tag{26}$$

The linear regression coefficient R is as follows:

$$R = L_{xy}/\sqrt{L_{xx}L_{yy}}, \tag{27}$$

where

$$\begin{aligned} &\bar{X} = \frac{1}{n}\sum X_i, \qquad \bar{Y} == \frac{1}{n}\sum Y_i, \\ &U_{xy} = \frac{1}{n}\sum X_I Y_i, \quad U_{xx} = \frac{1}{n}\sum X_i^2, \quad U_{yy} = \frac{1}{n}\sum Y_i^2, \\ &L_{xy} = U_{xy} - \bar{X}\bar{Y}, \quad L_{xx} = U_{xx} - \bar{X}^2, \quad L_{yy} = U_{yy} - \bar{Y}^2 \end{aligned} \tag{28}$$

and so the values of β and η are as follows:

$$\beta = -B \quad \eta = \exp\left(\frac{A}{\beta}\right). \tag{29}$$

In order to determine T_{st} and d_∞, an iterative procedure is proposed. Let us assume

$$d_\infty = d_{max} + \Delta d, \tag{30}$$

where d_{max} is the maximum corrosion wear in the given database and Δd is a small increment subjectively chosen. For example, one can choose $\Delta d = d_{max}/100$. For a given d_∞, an optimal value of T_{st} is defined to satisfy the condition of $\mathrm{d}R/\mathrm{d}t = 0$. That is, T_{st} can be determined from the following equation:

$$\left(\sum\frac{X_i}{t_i - T_{st}} - \sum\frac{\bar{X}_i}{t_i - T_{st}}\right)/L_{xx} - \left(\sum\frac{Y_i}{t_i - T_{st}} - \sum\frac{\bar{Y}_i}{t_i - T_{st}}\right)/L_{xy} = 0. \tag{31}$$

Assuming that

$$d_\infty(i+1) = d_\infty(i) + \Delta d \tag{32}$$

and compute the corresponding values of $T_{st}(i+1), R(i+1)$. If $R(i) < R(i+1)$, then using Eq. (32) to continue the loop until $R(i) > R(i+1)$. Then using Eqs. (26–29), we can obtainβ, η.

5.2. Method to determine four random model parameters

For corrosion databases, there are large uncertainties. It might be better to treat the four model parameters $d_\infty, T_{st}, \eta, \beta$ as random variables. In such a situation, the following method is proposed to determine the statistical characteristics of the four parameters.

Let us assume that $D_i(t_j)$ are the measured corrosion wear of a particular plate and $d(t_j)$ is the computed value of the corrosion wear by the new corrosion model, we can compute the following error functions:

$$\begin{aligned} Error1 &= \sum(D_i(t_j) - d(t_j))^2 \\ &= \sum\left(D_i(t_j) - d_\infty\left(1 - \exp\left(-\left(\frac{t_j - T_{st}}{\eta}\right)^\beta\right)\right)\right)^2. \end{aligned} \tag{33}$$

The minimization of Eq. (33) is difficult to perform and instead we use the following error function for minimization:

$$Error2 = \sum\left(\left(E(D_i(t_j)) - E\left(d_\infty\left(1-\exp\left(-\left(\frac{t_j - T_{\mathrm{st}}}{\eta}\right)^\beta\right)\right)\right)\right)^2 + \left(S(D_i(t_j)) - S\left(d_\infty\left(1-\exp\left(-\left(\frac{t_j - T_{\mathrm{st}}}{\eta}\right)^\beta\right)\right)\right)\right)^2\right) \tag{34}$$

where $E(\ldots)$ is the mean value of the random variable in parentheses and $S(\ldots.)$ is its standard deviation.

If all the parameters $d_\infty, T_{\mathrm{st}}, \eta, \beta$ are assumed to be normal random variables, the mean and standard deviation of a function of random variables can be calculated using a fast method proposed in [30]. Then by minimizing the error function using IMSL Fortran 90 MP library, the arguments of all the four parameters can be determined.

5.3. Example calculation and comparison with other corrosion models

Unfortunately, there is no actual corrosion database available to us at the moment. In order for comparison, we assume the following corrosion data given in Table 2 which simulate the continual measurements of corrosion wear on a sample of steel plates with paint coatings immersed in sea water. This type of measured data would reflect the actual corrosion mechanism and will have much less scatter than those measured from various existing ships (e.g. [26, 28]).

First let us assume that all the model parameters are deterministic and using the approach introduced in Section 5.1 to determine the four model parameters. The results are: $T_{\mathrm{st}} = 1.38$ years, $\eta = 9.19$, $\beta = 1.99$, $d_\infty = 1.64$ mm. The linear regression coefficient is $R = 0.998$. So the corrosion rate and the corrosion wear can be represented, respectively, by

$$r(t) = \begin{cases} 0, & 0 \leqslant t \leqslant 1.38, \\ 0.355\left(\dfrac{t-1.38}{9.19}\right)^{0.99} \exp\left\{-\left(\dfrac{t-1.38}{9.19}\right)^{1.99}\right\}, & 1.38 \leqslant t \leqslant t_L, \end{cases} \tag{35}$$

$$d(t) = \begin{cases} 0, & 0 \leqslant t \leqslant 1.38, \\ 1.64\left\{1 - \exp\left[-\left(\dfrac{t-1.38}{9.19}\right)^{1.99}\right]\right\}, & 1.38 \leqslant t \leqslant T_L. \end{cases} \tag{36}$$

According to Eqs. (17) and (18), the moment at which the maximum corrosion rate will be achieved is $T_A = 7.88$ years and the maximum corrosion rate is $r_{\max} = 0.153$ mm/year.

Using the same approach, the model parameters defined in Paik's model and Guedes Soares' model can also be determined in a deterministic sense. The fit of the

Table 2
Corrosion data assumed

t (years)	Mean d (mm)	Standard deviation	t (years)	Mean d (mm)	Standard deviation	t (years)	Mean d (mm)	Standard deviation
2.0	0.01	0.005	7.4	0.53	0.02	16.0	1.53	0.05
3.0	0.04	0.005	7.8	0.65	0.02	17.0	1.56	0.05
4.0	0.12	0.005	8.0	0.69	0.02	18.0	1.59	0.08
4.4	0.16	0.008	8.4	0.75	0.03	19.0	1.60	0.08
4.8	0.21	0.008	9.0	0.85	0.03	20.0	1.61	0.08
5.0	0.23	0.008	9.4	0.91	0.03	21.0	1.62	0.1
5.4	0.28	0.008	10.0	0.99	0.04	22.0	1.62	0.1
5.8	0.34	0.01	11.0	1.13	0.04	23.0	1.62	0.1
6.0	0.37	0.01	12.0	1.25	0.04	24.0	1.63	0.1
6.4	0.43	0.01	13.0	1.35	0.04	25.0	1.63	0.1
6.8	0.49	0.02	14.0	1.42	0.04			
7.0	0.53	0.02	15.0	1.49	0.05			

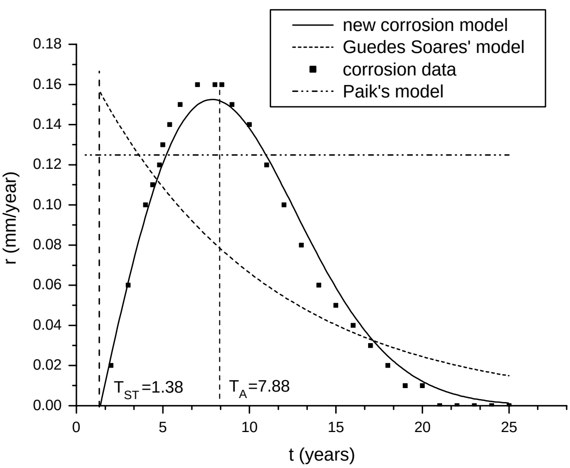

Fig. 5. Comparison of the corrosion rate for three corrosion models.

three models to the measured values is compared in Figs. 5 and 6, respectively, for corrosion rate and corrosion wear. The corrosion rate in Fig. 5 for the measured data is calculated using mean values for every time interval.

From Figs. 5 and 6, it can be seen that the present model provides the best fit. From the present corrosion model, the corrosion begins to occur at 1.38 years and from 1.38 to 7.88 years the corrosion rate accelerates and after 7.88 years it decreases because of the increasing thickness of the corrosion products.

Using Eq. (25), we can also plot the linear curve between X and Y and these are shown in Fig. 7 for the present model and Guedes Soares' model and in Fig. 8 for Paik's model. The same conclusion is obtained.

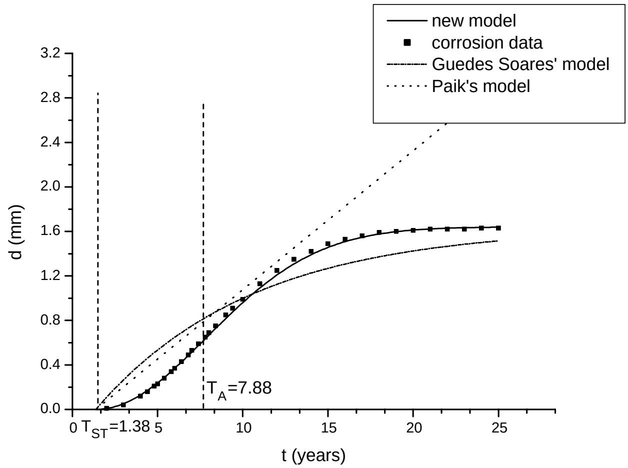

Fig. 6. Comparison of the corrosion wear for three models.

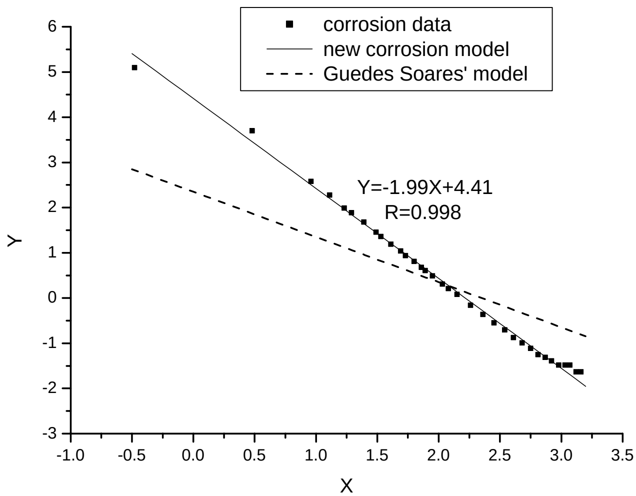

Fig. 7. The linear relationship between X and Y in Guedes Soares' model and the present model.

Now if all the model parameters $d_\infty, T_{\mathrm{st}}, \eta, \beta$ are taken as random variables, by applying the method presented in Section 5.2, the mean and standard deviation of the model parameters can be determined. The results are given in Table 3.

For Paik's model, using the same corrosion data (Table 2) to fit the corrosion model, Eq. (11) [26], the mean and standard deviation of coefficient c_1 can be determined and they are $\mu_{c1} = 0.1249$, $\sigma_{c1} = 0.0313$.

The linear relationship is

$$Y = 0.7201X + 1.6488 \tag{37}$$

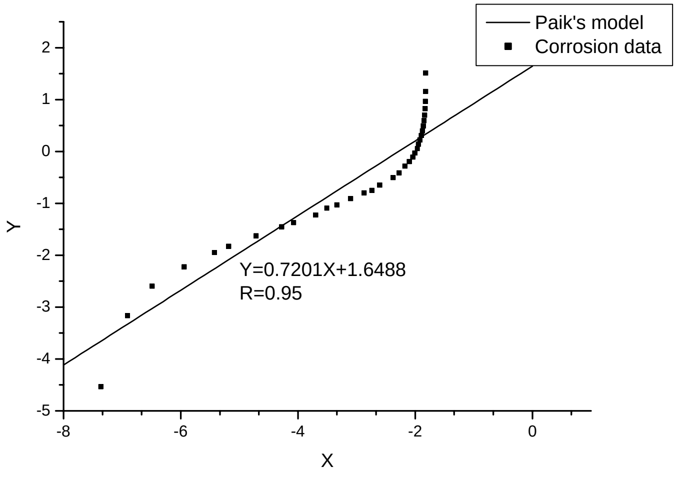

Fig. 8. The relationship of X–Y in Paik corrosion model.

Table 3
Statistical values of random variables of three corrosion models

Model style and distribution		Statistical values of random variables		
New model	Random variable distribution	Random variables	mean	Standard deviation
	Normal distribution	d_∞ (mm)	1.67	0.0674
		T_{st} (year)	1.40	0.0001
		β	1.97	0.0294
		η	9.15	0.0181
Guedes Soares' model	Normal distribution	d_∞ (mm)	2.28	0.0940
		T_{st} (year)	1.99	0.0001
		η	15.00	0.0001
		β	1.0	0.0000
Paik's model	Weibull distribution	C_1 ($C_2 = 1$)	w	k
			0.1013	0.7201

with a linear regression coefficient $R = 0.95$. In Eq. (37), $X = \ln x$; $Y = \ln[-\ln(1 - F_{c1}(x))]$. The relationship between X–Y and the measured corrosion data is shown in Fig. 8.

For Guedes Soares' model, the same curve fit approach as the new corrosion model is used and the results are also given in Table 3.

Obviously, the comparison made in this section is subjected to a deficiency. That is, the corrosion data is not actual, especially the data did show an accelerating phase and a decelerating phase which is the advantage of the new model. However, from the discussion on the corrosion mechanism, it is the authors' belief that the actual corrosion process might have three stages: (1) no corrosion, $t \in [0, T_{st}]$; (2) corrosion accelerating, $t \in [T_{\text{st}}, T_A]$; (3) corrosion decelerating, $t \in [T_A, T_L]$. The other two models compared do not have the capability to capture this feature. On the other hand, even if the actual data do not show accelerating and decelerating phases clearly and simply an arbitrary corrosion rate, based on the discussion of Section 4, the present model would also have a better curve-fitting ability because the other models are only specific cases of the new corrosion model. Of course, comparisons using many actual measurements of the corrosion data are required.

6. Time-dependent reliability analysis of a steel plated element

Unstiffened plates are the main structural components in ships and many other structures. Let us use such a simple plated element to compare the present corrosion model with those proposed by Paik et al. [26] and Guedes Soares and Garbartov [14]. It is assumed that the plate element is subjected to uniaxial compression. The limit state function can be expressed as

$$G(t) = \sigma_{u(t)} - \sigma_{\text{xav}}, \tag{38}$$

where $\sigma_{u(t)}$ is the ultimate strength at the time t, and σ_{xav} is the applied longitudinal compressive stress.

The ultimate strength of a plate element without considering the effects of initial deflection and residual stresses can be calculated by [31]:

$$\sigma_u/\sigma_y = \begin{cases} 1 & \text{if } \lambda \leqslant 1.9, \\ 0.08 + \dfrac{1.09}{\lambda} + \dfrac{1.26}{\lambda^2} & \text{if } \lambda > 1.9, \end{cases} \tag{39}$$

where $\lambda = (b/h(t))\sqrt{\sigma_y/E}$, $h(t)$ is the thickness of the plate at time t; b is the width of the plate.

The net thickness of the plate considering corrosion is

$$h(t) = h_0 - d(t). \tag{40}$$

The statistical characteristics related to corrosion wear $d(t)$ is given in Table 3. For other variables involved in the limit state function, Eq. (38), the statistical characteristics are given in Table 4.

Using the importance sampling method implemented in ISPUD program [32], the probability of failure and the corresponding reliability index of the steel plated element can be calculated at different times and the results are shown in Fig. 9. The corresponding results from other two models are also compared in this figure.

From Fig. 9, it can be seen that the corrosion models would have a quite significant influence on the reliability of structures even under the same loading and corrosive environment. This indicates that the choice of an appropriate corrosion model is important to analyze the reliability of an existing structure.

Table 4
Statistical characteristics of random variables

Variable	Mean	Standard deviation	Distribution type
σ_y (MPa)	250	15	Lognormal
E (MPa)	200,000	4000	Normal
b (mm)	700	10	Normal
σ_{xav} (MPa)	100.0	20	Type I extreme
h_0 (mm)	11	0.55	Normal

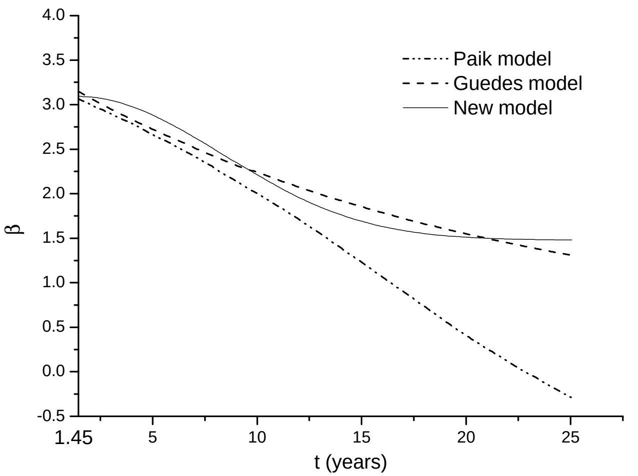

Fig. 9. Reliability index of the steel plated element by three models.

7. Summary and conclusions

Reliability assessment by considering corrosion and fatigue is very important for the optimal scheme of maintenance and repair. Different corrosion models would have significant influence on the assessed reliability. By studying some of the currently available corrosion models, it is argued in this paper that these models may not fully reflect the reality. A new corrosion model is proposed to improve this situation. This model is compared with two representative existing corrosion models and the effect of corrosion models on the time-dependent reliability is also studied using a steel plated element. Through this study, the following conclusions can be drawn:

1. In the new model, the whole corrosion process is divided into three stages: no corrosion, corrosion accelerating, corrosion decelerating. There does not exist any sudden jump in the corrosion rate for metals with CPS which might be better to reflect the real corrosion process.
2. The new corrosion model is more flexible and can accommodate other existing corrosion models as its special cases. Thus, the new model would have a better curve-fitting capability.

3. Using an assumed corrosion database, the flexibility and accuracy have been briefly demonstrated and the influence of the corrosion models on reliability is confirmed. Obviously the comparison based on the actual measurements are required for sound conclusion.

References

[1] Emi H, Kumano A, Baba N, Yamamoto N, Nakamura Y, Shihara H. A study on life assessment of ships and offshore structures (Part 1: basic study). J Soc Nav Archit Jpn 1991;169: 443–54.
[2] Emi H, Yuasa M, Kumano A, Kumamoto N, Yamamoto N, Matsunaga M. A study on life assessment of ships and offshore structures (2nd report: risk assessment of fatigue failures of hull structures). J Soc Nav Archit Jpn 1992;172:627–35.
[3] Emi H, Yuasa M, Kumano A, Arima T, Yamamoto N, Umino M. A study on life assessment of ships and offshore structures (3rd report: corrosion control and condition evaluation for a long life service of the ship). J Soc Nav Archit Jpn 1993;174:735–44.
[4] RINA. Proceedings of the International Conference on Design, Operation of Bulk Carriers. London: The Royal Institution of Naval Architects, 1998.
[5] TSCF. Condition evaluation, maintenance of tank structures. Witherby, London: Tanker Structure Co-operative Forum, 1992.
[6] TSCF. Guidance manual for tank structures. Witherby, London: Tanker Structure Co-operative Forum, 1997.
[7] Gardiner CP, Melchers RE. Corrosion analysis of bulk carrier ships. In: Proceedings of the Conference on Corrosion and Prevention, 9–12 November, Brisbane, 1997.
[8] Guedes Soares C, Garbatov Y. Reliability of maintained ship hulls subjected to corrosion. J Ship Res 1996;40(3):235–43.
[9] Guedes Soares C, Garbatov Y. Reliability assessment of maintained ship hulls with correlated corroded elements. Mar Struct 1997;10:629–53.
[10] Guedes Soares C, Garbatov Y. Reliability of maintained ship hull girders subjected to corrosion and fatigue. Struct Saf 1998;20:201–19.
[11] Guedes Soares C, Garbatov Y. Reliability of plate elements subjected to compressive loads and accounting for corrosion and repair. In: Shiraishi N, Shinozuka M, Wen YK, editors. Structural safety and reliability, vol. 3. Rotterdam: Balkema, 1998. p. 2013–20.
[12] Guedes Soares C, Garbatov Y. Reliability of corrosion protected and maintained ship hulls subjected to corrosion and fatigue. J Ship Res 1998;43(2):65–78.
[13] Guedes Soares C, Garbatov Y. Reliability of maintained ship hulls subjected to corrosion and fatigue under combined loading. J Constr Steel Res 1999;52:93–115.
[14] Guedes Soares C, Garbatov Y. Reliability of maintained, corrosion protected plates subjected to non-linear corrosion and compressive loads. Mar Struct 1999;12:425–45.
[15] Sun HH, Bai Y. Reliability assessment of a FPSO hull girder subjected to degradations of corrosion and fatigue. In: Proceedings of the Tenth International Offshore and Polar Engineering Conference, May 28–June 2, Seattle, USA, 2000. p. 355–63.
[16] Sun HH, Bai Y. Time-variant reliability of FPSO hulls. Transactions of SNAME, 2001.
[17] Wirsching PH, Ferensic J, Thayamballi A. Reliability with respect to ultimate strength of a corroding ship hull. Mar Struct 1997;10:501–18.
[18] Guedes Soares C, Ivanov LD. Time-dependent reliability of the primary ship structure. Reliability Eng Syst Saf 1989;26:59–71.
[19] Boyd WK, Fink FW. Corrosion of metals in marine environments—an overview. In: Schumacher M, editor. Seawater corrosion handbook. New Jersey: Noyes Data Corporation, 1979. p. 1–104.
[20] Melchers RE. Probabilistic modelling of marine corrosion of steel specimens. In: Proceedings of the ISOPE'95, 12–15 June, the Hague, the Netherlands, 1995. p. 204–10.

[21] Melchers RE. Modeling of marine corrosion of steel specimens. In: Kain RM, Young WT, editors. Corrosion testing in natural waters, vol. 2. Philadelphia: ASTM STP 1300, 1997. p. 20–33.
[22] Melchers RE. Probabilistic modelling of immersion marine corrosion. In: Shiraishi N, Shinozuka M, Wen YK, editors. Structural safety and reliability, vol.3. Rotterdam: Balkema, 1998. p. 1143–9.
[23] Melchers RE. Corrosion uncertainty modelling for steel structures. J Constructional Steel Res 1999;52:3–19.
[24] Southwell CR, Bultman JD, Hummer Jr CW. Estimating of service life of steel in seawater. In: Schumacher M, editor. Seawater corrosion handbook. New Jersey: Noyes Data Corporation, 1979. p. 374–87.
[25] Loseth R, Sekkeseter G, Valsgard S. Economics of high tensile steel in ship hulls. Mar Struct 1994;7:31–50.
[26] Paik JK, Kim SK, Lee SK. Probabilistic corrosion rate estimation model for longitudinal strength members of bulk carriers. Ocean Eng 1998;25(10):837–60.
[27] Yamamoto N, Kumano A, Matoba M. Effect of corrosion and its protection on hull strength (2nd report). J Soc Nav Archit Jpn 1994;176:281–9.
[28] Yamamoto N, Ikegami K. A study on the degradation of coating and corrosion of ship's hull based on the probabilistic approach. Proceedings of the International Offshore Mechanics and Arctic Engineering Symposium (OMAE'96), vol. 2. 1996. p. 159–66.
[29] Yamamoto N. Reliability based criteria for measures to corrosion. In: Proceedings of the 17th International Conference on Offshore Mechanics and Arctic Engineering (OMAE'98), Safety and Reliability Symposium, New York, USA: ASME, 1998.
[30] Cui WC, Xu XD, Qiu Q. A fast method to calculate the mean and standard deviation of the funtion of random variables. J Ship Mech 1998;2(6):50–60 [in Chinese].
[31] Cui WC, Mansour AE. Effects of welding distortions and residual stresses on the ultimate strength of long rectangular plates under uniaxial compression. Mar Struct 1998;11:251–69.
[32] Bourgund V, Bucher CG. ISPUD—importance sampling procedure using design points—a user's manual. Report 8–86, Institute of Engineering Mechanics, University of Innsbruck, 1986.